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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 2 6 3 4 3 8 9 6 3 5 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the equation $8^{3-6x} = 4 \times 5^{-2x}$. Give your answer correct to 3 decimal places. [4]

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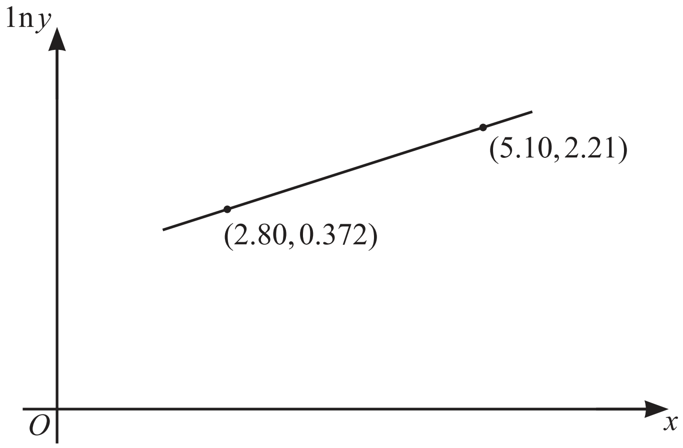
2 Find the exact coordinates of the stationary point of the curve $y = e^{2x}\sin 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [5]

3 The square roots of $24-7\mathrm{i}$ can be expressed in the Cartesian form $x+\mathrm{i}y$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $24-7\mathrm{i}$ in exact Cartesian form. [5]

4



The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c. Give each value correct to 2 significant figures. [4]

5 Express $\dfrac{6x^2-2x+2}{(x-1)(2x+1)}$ in partial fractions. [5]

6 **(a)** On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z-4-3\text{i}| \leqslant 2$ and $\arg(z-2-\text{i}) \geqslant \frac{1}{3}\pi$. [5]

(b) Calculate the greatest value of $\arg z$ for points in this region. [2]

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7 Let $f(x) = 8x^3 + 54x^2 - 17x - 21$.

(a) Show that $x + 7$ is a factor of $f(x)$. [1]

(b) Find the quotient when $f(x)$ is divided by $x + 7$. [2]

(c) Hence solve the equation

$$8\cos^3\theta + 54\cos^2\theta - 17\cos\theta - 21 = 0,$$

for $0° \leqslant \theta \leqslant 360°$. [3]

8 **(a)** Express $3\cos 2x - \sqrt{3}\sin 2x$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and α. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{3}{\left(3\cos 2x - \sqrt{3}\sin 2x\right)^2}\,dx$, simplifying your answer. [5]

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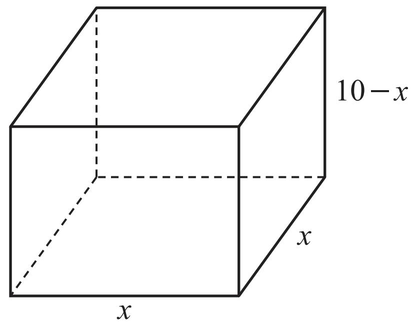

A container in the shape of a cuboid has a square base of side x and a height of $(10-x)$. It is given that x varies with time, t, where $t > 0$. The container decreases in volume at a rate which is inversely proportional to t.

When $t = \frac{1}{10}$, $x = \frac{1}{2}$ and the rate of decrease of x is $\frac{20}{37}$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x - 3x^2)}.$$ [5]

(b) Solve the differential equation, obtaining an expression for t in terms of x. [6]

10 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + 2a\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}),$$

where a is a constant.

(a) Given that the acute angle between the directions of these lines is $\frac{1}{4}\pi$, find the possible values of a. [6]

(b) Given instead that the lines intersect, find the value of a and the position vector of the point of intersection. [5]

11 Use the substitution $2x = \tan\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{12}{\left(1+4x^2\right)^2}\,\mathrm{d}x\ .$$

Give your answer in the form $a + b\pi$, where a and b are rational numbers. [9]

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Additional page

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